QP CODE: 19102565

Reg No : .....

Name : .....

BSc DEGREE (CBCS) EXAMINATION, OCTOBER 2019

Fifth Semester

Core Course - MM5CRT01 - MATHEMATICAL ANALYSIS

B.Sc Computer Applications Model III Triple Main ,B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

819A8253

Maximum Marks: 80 Time: 3 Hours

Part A

*Answer any **ten** questions.*

Each question carries 2 marks.

1. Let $S = \{1, 2\}$ and $T = \{a, b, c\}$ then determine all the injections from S to T ?
2. Define the notion of inequlity between two real numbers in terms of the positive set P in the ordering property of real numbers?
3. Define bounded and unbounded sets with proper examples?
4. Define rational numbers in terms of the decimal expansion? Find the decimal representation of $-\frac{2}{7}$?
5. If $0 < b < 1$, prove that $lim(b^n) = 0$.
6. Define bounded sequence. Give an example.
7. Find the limit of $(3n^{1/2n})$.
8. Define Cauchy sequence. Give an example.
9. Let (x_n) and (y_n) be two sequences of real numbers and suppose that $x_n \leq y_n$ for all n. Prove that if $\lim x_n = +\infty$ then $\lim y_n = +\infty$.
10. State the Limit Comparison Test for the series.
11. Is the series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n}$ is absolutely convergent or not? Why?
12. Show that $\lim_{x \to c} x = c$ for any $c \in \mathbb{R}$.

(10×2=20)

Part B

Answer any six questions.

Each question carries 5 marks.

13. Show that for all $a, b \in R$

    (a.) $\max\{a,b\} = \frac{1}{2}(a+b+|a-b|)$

    (b.)$\min\{a,b\} = \frac{1}{2}(a+b-|a-b|)$

14. Prove that If A, B are bounded sets then $Sup\,(A+B) = Sup\,A + Sup\,B$ where $A+B = \{a+b : a \in A, b \in B\}$

15. Prove that $\lim (n^{1/n}) = 1$.

16. Prove that $lim(\frac{sinn}{n}) = 0$.

17. State and prove Monotone Convergence Theorem.

18. State and prove the root test for the absolute convergence of a series in R.

19. State and prove Abel's Lemma.

20. Prove that $\lim_{x\to 0} \cos(\frac{1}{x})$ does not exist but that $\lim_{x\to 0} x\cos(\frac{1}{x}) = 0$.

21. Check whether the one-sided limits of the function $g(x) = e^{\frac{1}{x}}$ at $x = 0$ exist or not.

(6×5=30)

Part C

Answer any two questions.

Each question carries 15 marks.

22. (a.) State and Prove Nested interval property?

    (b.) Prove that the set of real numbers is not countable?

23. (a) State Monotone Convergence Theorem.

    (b) Prove that for any real number a > 0, there exists a sequence (s_n) of real numbers that converges to $\sqrt{a}$.

    (c) Use the above sequence to evaluate the value of $\sqrt{5}$ correct to 5 decimal places.

24. Test the convergence and absolute convergence of the following series.

    - $\sum_1^\infty \frac{(-1)^{n+1}}{(n+1)}$
    - Whose nth term is $\frac{(-1)^n n^n}{(n+1)^{n+1}}$

25. (a) Let $A \subseteq \mathscr{R}$, $f, g : A \to \mathscr{R}$, and let $c \in \mathscr{R}$ be a cluster point of A, Suppose that $f(x) \leq g(x)$ for all $x \in A$, $x \neq c$, Then prove the following

- If $\lim_{x \to c} f = \infty$, then $\lim_{x \to c} g = \infty$.
- If $\lim_{x \to c} g = -\infty$, then $\lim_{x \to c} f = -\infty$.

(b) Give an example of a function that has a right-hand limit but not a left-hand limit at a point.

(c) Evaluate the limit or show that it do not exist " $\lim_{x \to 1} \frac{x}{x-1}$ where $x \neq 1$.

(2×15=30)